Sam Lazrak

CS 303-2D Algorithms and Data Structures

Homework Assignment 2

1/23/18

1. Exercise 2.2-1: Express the function n3/1000 -100n2 – 100n + 3 in terms of -notation.
2. Answer: (n3)
3. Exercise 2.2:
4. Answer: We need to prove that all of the elements in A[1], A[2], ….., A[n] are in A’[1], A’[2], …., A’[n].
5. State and prove loop invariant.

Answer:

Loop Invariant: In each iteration, the subarray A[j…n] contains all of the elements that were originally in A[j…n] and the first element of the array is the smallest.

Initialization: The loop invariant holds because initially, the array A[j…n] contains only the last element A[n] and it is the smallest.

Maintenance: In each iteration, A[j] is compared with A[j-1] and swapped if A[j] is smaller than A[j-1]. The loop invariant holds because only one element that was contained in A[j…n] is added through each iteration and if swapped, the first element of the array is the smallest.

Termination: The loop terminates at j = i+1 which shows that all of the elements that were originally in A[i] are in A[j] and A[i] is the smallest element.

1. Using the previous termination condition state a loop invariant for the for loop in lines 1-4 that proves inequality.

Answer:

Loop invariant: In each iteration, the array A[1…i-1] has all of elements sorted such that they are smaller than or equal to the elements in the array A[i…n].

Initialization: At the start, the array A[1…i-1] is empty thus the loop invariant holds.

Maintenance: After the inner loop terminates, all of the elements of A[i…n] are sorted such that A[i] is the smallest element of the array. Thus, all the elements of A[1…i-1] contain all of the elements in sorted order from array A[i…n] and so when the outer loop is terminated, the loop invariant will hold.

Termination: The loop terminates when i is equal to the length of the array so at that point, every element in the array is in sorted order.

1. Indicate, for each pair of expressions (A, B) in the table below, whether A is O, o, , or‚ of B. Assume that k 1, > 0, and c > 1 are constants. Your answer should be in the form of the table with “yes” or “no” written in each box.

Answer:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| A | B | O | o |  |  |  |
|  |  | yes | yes | no | no | no |
|  |  | yes | yes | no | no | no |
|  |  | no | no | no | no | no |
|  |  | no | no | yes | yes | no |
|  |  | yes | no | yes | no | yes |
|  |  | yes | no | yes | no | yes |